Modeling Copper Clad Steel from First Principles

Edward Harriman
AE6TY
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Preface

This paper is a snapshot. It conveys the present state of an incomplete and ongoing development effort. However, initial results are promising and consistent enough to be useful in understanding the subject of low frequency or bimetal coaxial and twin lead transmission lines.

The Stage

Up until May, 2019, SimSmith provided two basic transmission line models: the 'Simplified' model and the k0k1k2 model. The Simplified model takes a single loss parameter 'loss' along with a frequency 'Fo' and computes the loss for any frequency using the simple equation:

Computed_loss ~= loss*Sqrt(F/Fo)

The k0k1k2 model uses the model developed by Howard Johnson & Martin Graham. It takes three parameters called k0, k1, and k2. The interpretation of these parameters is discussed below.

The 'Simplified' model is adequate when operating close to the specified loss/frequency values given. The 'k0k1k2' model is superior in every way and covers generally frequency ranges from several hundred kilohertz up to the several hundred-megahertz range under most conditions. This frequency range covers nearly everything of interest to the author.

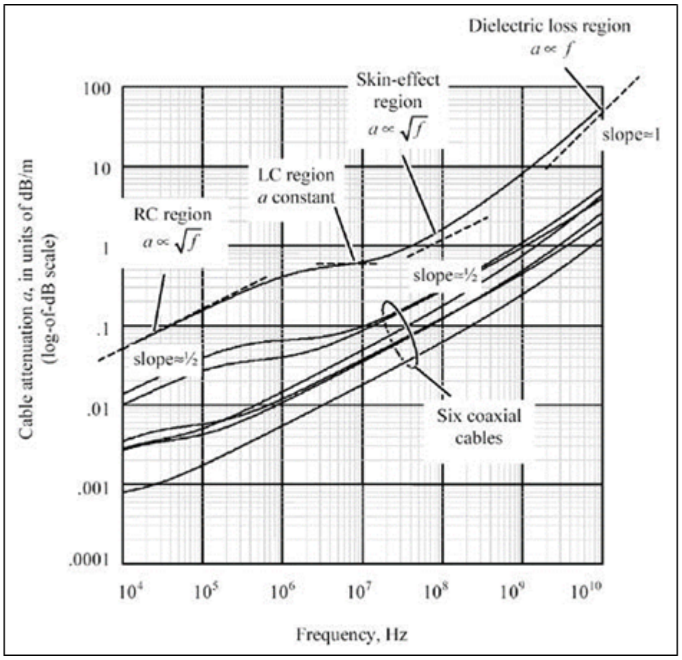
There are, however, two areas where the k0k1k2 model may prove inadequate. Specifically, when the coax center conductor is constructed from a 'clad' material OR when the frequency range is below a few hundred kilohertz. This paper describes the development of a model which can cover these two areas.

An important goal of this effort was the creation of a model that could predict coax and twin lead transmission line characteristics from physical properties. This effort is ongoing.

Background: the k0k1k2 model

The underlying principles of the k0k1k2 model is described by Johnson in chapter 2. Johnson does a mathematical derivation which Dan Maguire (AC6LA) has implemented. Dan's implementation, written for Excel, was ported to SimSmith as an internal java routine. Dan has also ported the algorithms to SimSmith's Anvil programming language.

The basic idea of the algorithm is that there are three regions of operation. Dan was kind enough to upload a page from Johnson, Figure 3.1.



Belden cable type	Class	Conductor composition
8216	RG-174/U	Bare copper-plated steel
84316	RG-316/U	Copper-plated steel with silver coating
8259	RG-58A/U	Tinned copper
8240	RG-58/U	Bare copper
84303	RG-303/U	Copper-plated steel with silver coating
8237	RG-8/U	Bare copper

As can be seen in the chart above, there are four ‘regions’ of interest. The first is very low frequencies where the inductance of the center conductor does not come into play. This is called the ‘RC’ region. Here, the ‘k0’ factor dominates. As shown it the figure above, the power dissipated by an RC network grows like the square root of frequency for lower frequencies.

As the frequency increases, the inductance of the center conductor comes into play and the inductance starts to cancel out the capacitance. This is called the LC region and is shown in the figure. In this region the resistance is relatively constant, and the skin effect has not yet started to manifest. The loss graph enters a ‘level’ place where loss doesn’t change very much.

As the frequency increases further, the skin effect comes into play. In this region, resistance (and therefore loss) changes with the square root of frequency. This effect can be ‘scaled’ for different conductor configurations using the ‘k1’ factor.

At even higher frequencies, the dielectric losses can come into play. Here the losses grow linearly with frequency. This loss can be scaled using ‘k2’ and depends on the material used for the dielectric.

Of critical import is the blending of the effective resistance and conductive losses as the Johnson model transitions between the regions. The exact solution requires the use of Bessel functions which are computationally complex. Johnson provides a blending algorithm which is sufficiently accurate for most applications. The interested reader should ‘go to the source’ and read Johnson. It’s quite enlightening.

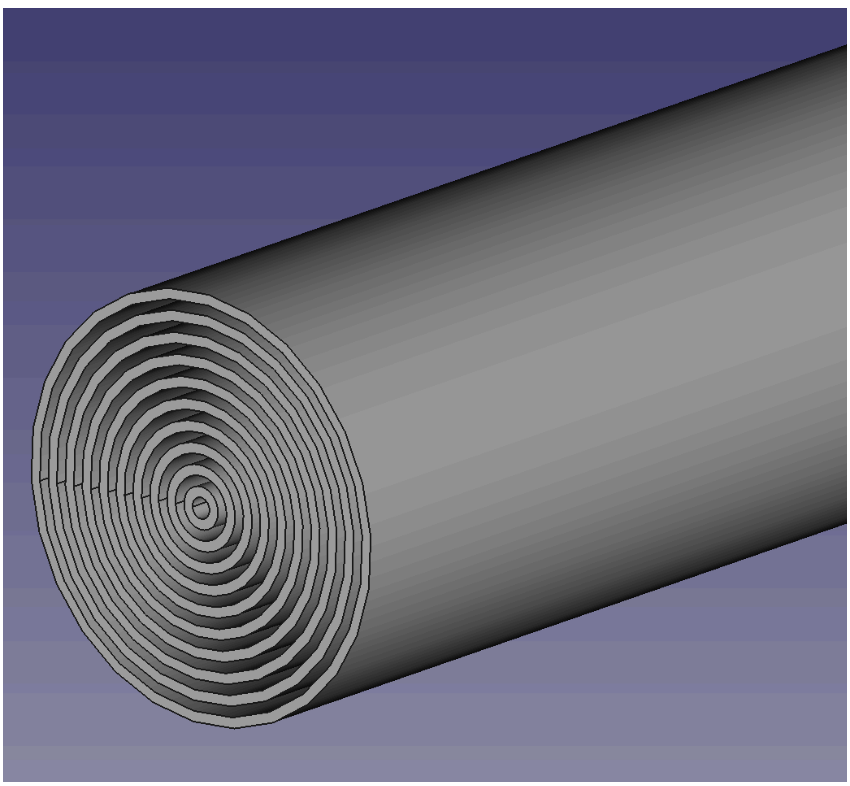
One significant shortcoming is that the k0k1k2 model requires the ‘fitting’ of the k values to specification or measurements. Generally, the manufacturers do not provide specifications below 1 MHz. Occasionally, not below 10 MHz. This reflects the manufacturer's intended operating region. Generally, manufacturer's specifications are sufficient to fit k values with enough precision to predict behavior well below 1 MHz; down to the 50 kHz range is quite common.

The Model's Fundamentals

The following is based on section 2.9 of "High-Speed Signal Propagation, Advanced Black Magic" by Howard Johnson and Martin Graham. Determining coax characteristics is done numerically from manufacturer provided geometric and material specifications. (With the exception of the dielectric loss specification.)

The following model uses only three well established laws: Ampere's law, Faraday's law, and Ohms law. While the inspiration comes from Johnson, this paper differs somewhat from that described by Johnson so as to incorporate the shield losses explicitly and so as to reduce the computational complexity at higher frequencies.

The basic idea of the model is that a coax cable center conductor can be divided up into a number of concentric tubes:

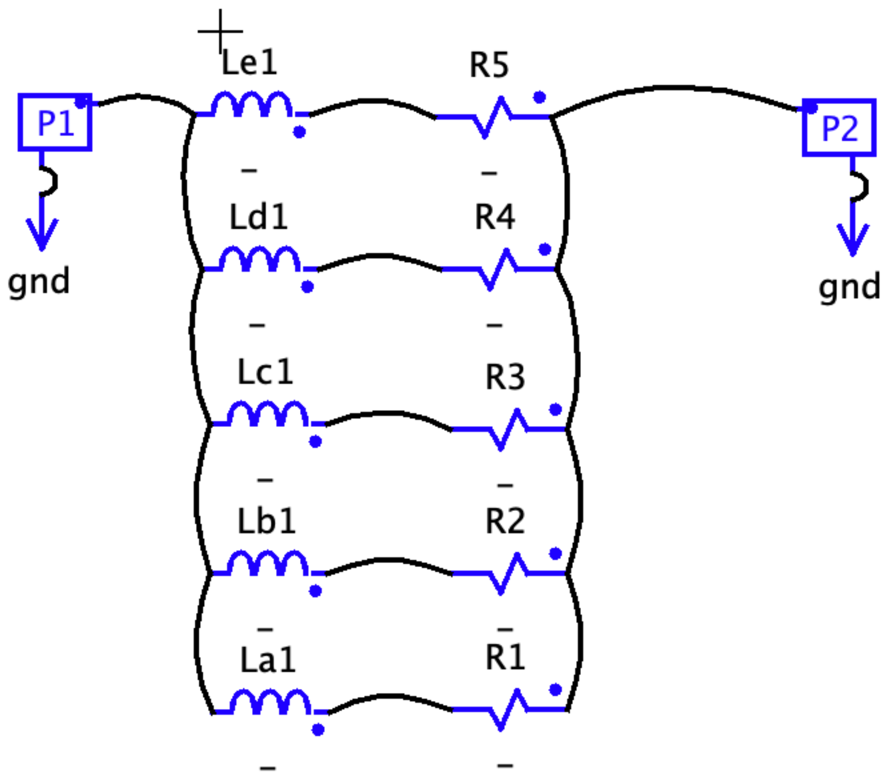


This model leverages the fact that the currents flowing in each of the tubes are 'parallel' and there is no current flowing between the tubes.   The tubes are all tied together at each end and the tubes are actually touching and not separated as shown in the image above.

The entire circuit can be represented as a discrete circuit of the form shown below. Note that there is a shield involved which is not shown. The presence of the shield allows the computation of each of the inductors using Ampere's law:

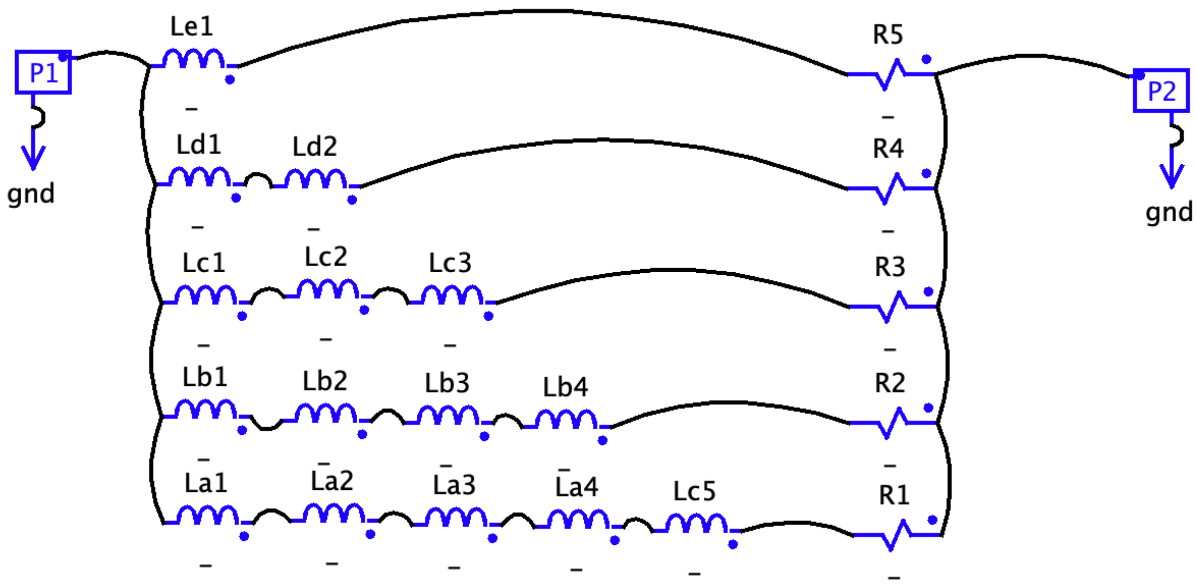
relMU * MUo * Length * Ln(Rshield/Ri)/(2*Pi)

where 'Ri' is the radius of a given tube and 'Rshield' is the radius of the shield.



This simple model already helps explain some behaviors. If the tubes all have the same thickness, then the outermost tube will have the lowest resistance because it has a larger conductive area. Further, inspection of Ampere's law will reveal that the induction of the outer tubes is lower. The combination of these two observations will lead to the conclusion that, as the frequency increases, current will tend to flow in the outermost tubes. This demonstrates the existence of the 'skin effect'.

Of course, the inductors are magnetically coupled since there is shared flux between the tubes and the shield. To understand how to compute the coupling coefficient, the inductors can be expanded to the following form:



Again, the inductor values can be computed using Ampere's law. It is important to note that

$$\mathrm{Ln}(C/A) = \mathrm{Ln}(C/B) + \mathrm{Ln}(B/A)$$

This means that the computation of each of the inductors can be determined from geometry alone: the leftmost inductors are all computed from the conductor diameter and the shield. All the other inductors are computed using the diameters of adjacent tubes. This observation comes into play when the permeability of a given ring is different than that of free space… for example, when a ring is constructed with steel.

Since the inductor columns represent the 'flux' in the space between them, the inductors in each column all have the same values. Further, since the inductors in a column share exactly the same flux, their coupling is perfect. Thus, all inductors in a given column have a coupling coefficient of 1. In spice and SimSmith, then, the coupling coefficients would be written:

```
K0 Le1 Ld1 Lc1 Lb1 La1 1;
K1 Ld2 Lc2 Lb2 La2 1;
K2 Lc3 Lb3 La3 1;
K3 Lb4 La4 1;
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The above circuit can be analyzed using SimSmith. Even the above simplified circuit can be used to do some exploration.

Rough Calculations

A few sanity checks may be in order. For example, if the resistance is low then the inductors will dominate. Since all tubes are coupled perfectly to the outermost tube, any voltage drop across this outer tube will generate an equivalent 'back EMF' on all the inner tubes. This means the inner tubes will have NO current flowing. Conclusion: Perfect conductors lead to zero skin depth.

Another thought experiment is that when the frequency goes to zero, the impedance would be dominated by all the resistors in parallel which is all the tubes in parallel which sum up to the whole center conductor. Conclusion: Zero frequency leads to a pure resistance.

Another: if the mu of the material increases then the inductances increase and so the current is pushed to the outside. Conclusion: increasing mu reduces the skin depth.

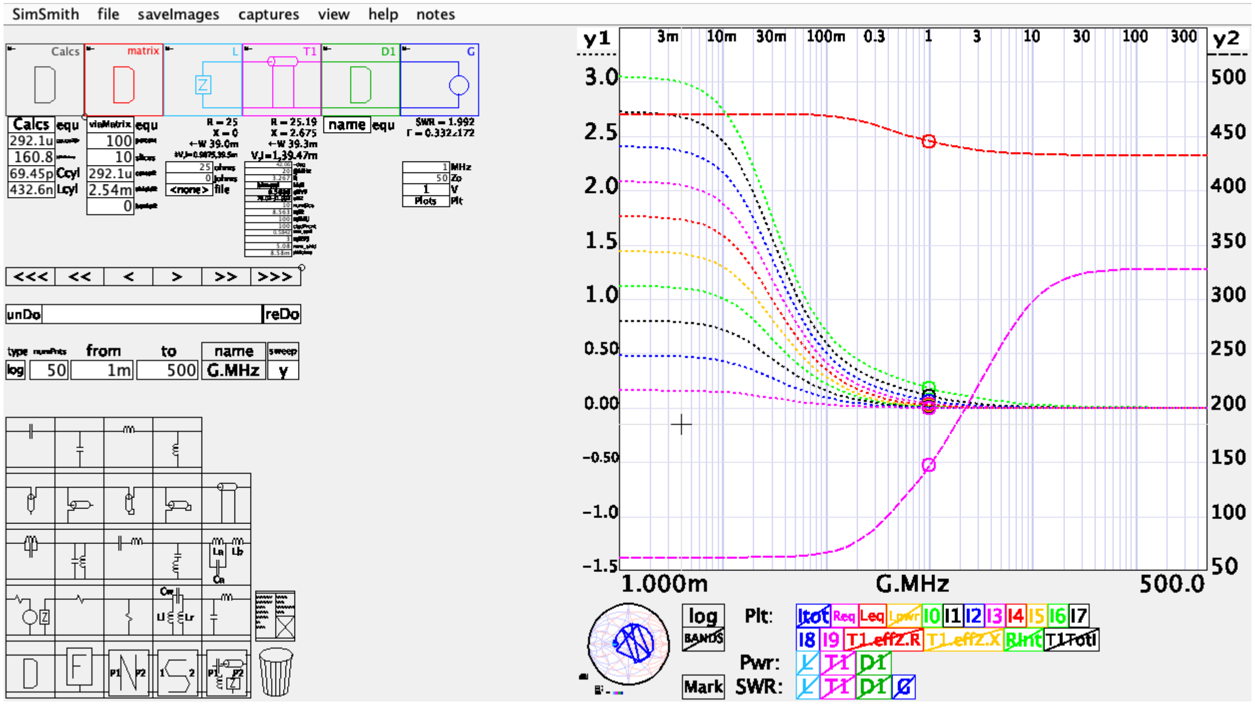
And: since the currents in the tubes see different inductances, their phases will be different. Currents on the inner 'tubes' will necessarily be delayed. When we plot the waves of these currents, they should show this phase delay.

The above circuit can also be used to predict some shortcomings of the model. First and foremost, when the frequency gets sufficiently high, all the current will pass through a single resistor BUT that resistor has a fixed value. All other models indicate that as the frequency increases the resistance also increases. At some point, regardless of how finely we divide up the conductor into smaller and smaller tubes, ultimately, the approximation must break down and the model will be inadequate.

Another shortcoming is that the above circuit does not discuss the material between the conductor and the shield. This material, the 'dielectric', will come into play when computing the impedance and velocity factor. Additionally, there is a frequency dependent dielectric loss which is not modeled above. We will return to these issues later in this paper.

Initial Results

The above circuit was expanded to include 10 tubes. The currents in each of the tubes was plotted against frequency along with the effective resistance and inductance:



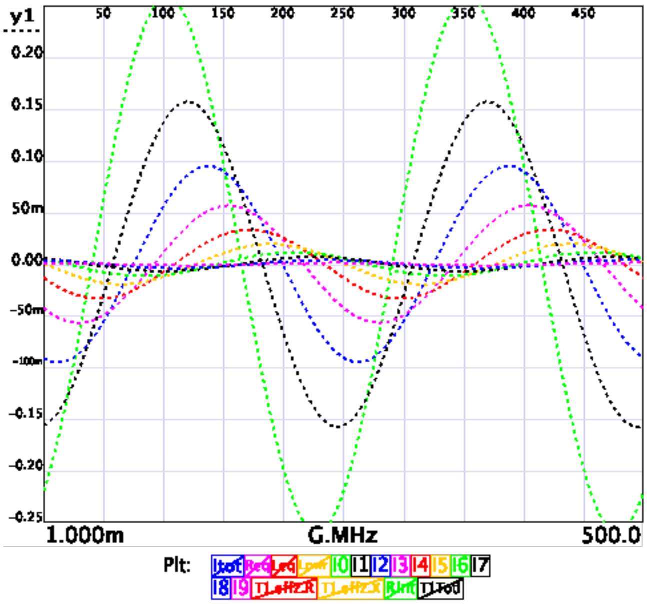
Notice that at low frequency (1 khz) the inductors have little effect and the current is evenly distributed throughout the conductor. Here, the tubes all have the same thickness and so the outer tubes have more cross-sectional area and therefore more current. Notice that as the frequency increases, the current in the inner tubes drops to near zero sooner than current in the outer tubes. Thus, one can see the ‘skin effect’ as a function of frequency.

Notice that the effective resistance (the magenta dashed line starting low and going higher) is relatively constant as the frequency starts to increase… the space where the inductances remain relatively low. As the frequency increases further and the current in the outer tubes finally starts to drop, the effective resistance increases.

As for the inductance (shown as dashed red), it starts relatively high. As frequency increases, the effective inductance declines (as the current is pushed to the outer tubes) and then reaches a minimum value.

Finally, as predicted, the effective resistance reaches a plateau where all the current is flowing in the outermost tube. Since this resistance is not frequency dependent the effective resistance doesn’t go down: one of our expected shortcomings is demonstrated.

Another prediction, the currents will be delayed in the inner tubes. This can be seen graphically in the following:



Thus, we see that the currents are acting as expected.

Problem Reduction

Unfortunately, using SimSmith to solve the given equations is computationally inefficient. For small numbers of tubes, things are tolerably quick. However, for large circuits, computation time becomes unbearable. One reason is that the simple circuit analysis is computing a large number of uninteresting node values. Specifically, the voltages across all the inductors. In the above circuit of 5 tubes, only 5 values are needed: the 5 currents flowing through the tubes. BUT… the SimSmith solution also solves for the 10 'internal voltage' nodes between the inductors. Thus, SImSmith will solve for 3 times as many variables as actually necessary. (As an aside, SImSmith is also very inefficient in creating the matrix as well making it a very bad choice, indeed.)

Remembering that matrix inversion takes O(N^3), solving for 3 times as many variables as necessary means 27 times the time… some reduction is highly desirable.

In essence, we want to strip down the matrix used by SimSmith. (I note that Spice could and quite possibly does do this stripping but I'm not privy to its internals. I know SImSmith does not do the following reduction automatically.) The condensed form of the matrix is as follows:

R1+wL1	wL1	wL1	wL1	wL1	wL1	= 1
wL1	R2+wL2	wL2	wL2	wL2	wL2	= 1
wL1	wL2	R3+wL3	wL3	wL3	wL3	= 1
wL1	wL2	wL3	R4+wL4	wL4	wL4	= 1
wL1	wL2	wL3	wL4	R5+wL5	wL5	= 1
wL1	wL2	wL3	wL4	wL5	R6+wL6	= 1

Note that the inductor values are frequency independent. This means that the computation of the inductor values need not be particularly efficient given the matrix solution takes O(N^3) time AND must be done for each frequency of interest. (I also note that there are probably optimizations which can be done given the matrix is symmetric… I'm not a mathematician and things are tolerably fast at this point). So, the primary step in supporting an efficient implementation of this algorithm was to simplify the matrix to reduce SimSmith overhead, and to 'pull out' SimSmith's internal matrix inversion logic. Both of these were done.

Simple Results

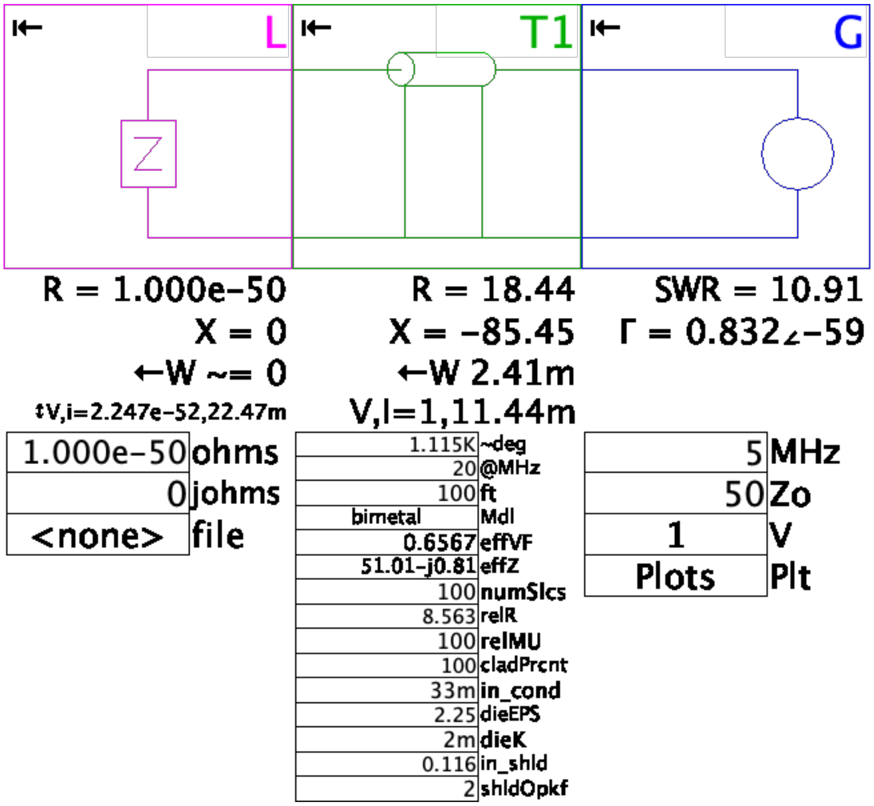
Coax specifications from various manufactures are not complete enough to do an exact 'from dimension' simulation. For example, they specify the material for the dielectric but don't really give things like 'relative Epsilon'. (I assume the relative permittivity for polyethylene is 2.25 as found on the web.) The first subject for analysis was Belden 9201 which is an RG58 type cable. As it turns out, this was a bad choice as there is some confusion about the loss of this coax in the 1 to 10 MHz range. Sorry… future versions of this paper will use a different coax.

Cond diameter:	.033 in	
Shield Diameter:	.116 in	
Relative perm	2.25	polyethylene

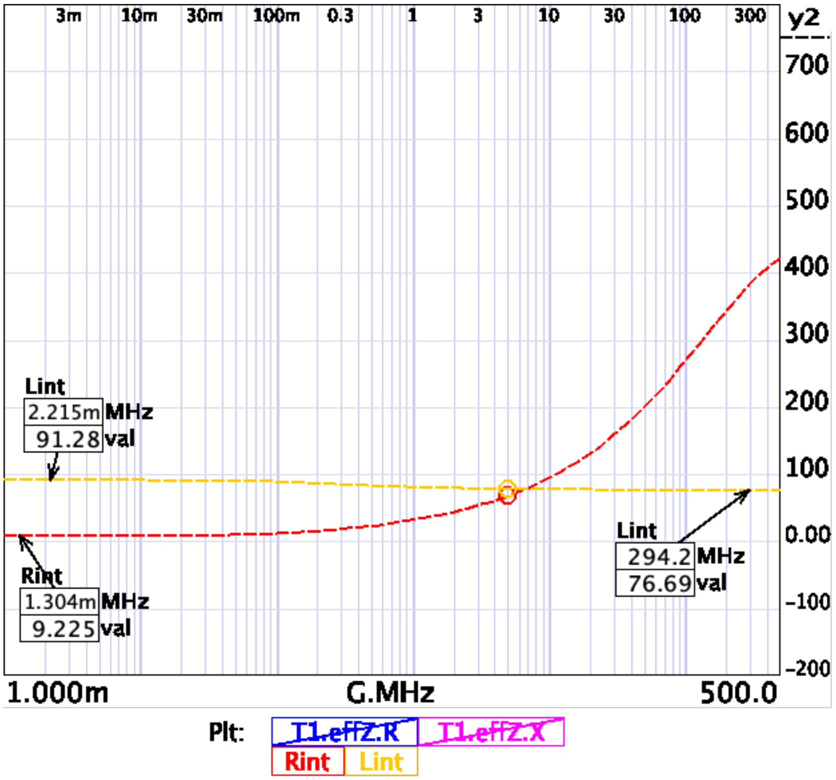
The 'bimetal' transmission line model takes the following parameters:

numSlcs	number of tubes
relR	the relative resistance of the 'steel'…
relMU	the relative permeability of the 'steel'…
cladPrcnt	the thickness of the cladding as a percentage of radius
in_cond	the diameter of the conductor in inches
dieEPS	the relative permittivity of the dielectric
dieK	the frequency dependent conductance of the dielectric
in_shld	the diameter of the shield in inches.
shldOpkf	the resistance of the shield in ohms per 1000 feet.

From just this we get:

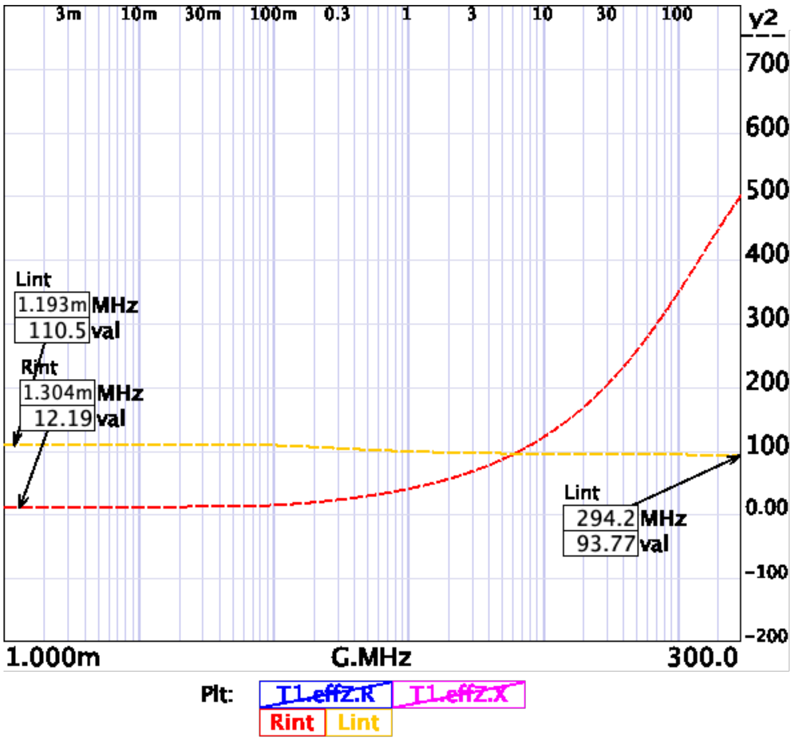


Notice that the computed 'effZ' is 51.01 – j.81 ohms. The nominal impedance from Belden indicates 52 ohms so the model is reasonably close. Further, the effective velocity factor is .6567… close to he expected .667. The dc resistance and high frequency inductances can be extracted by plotting them on the Square chart:

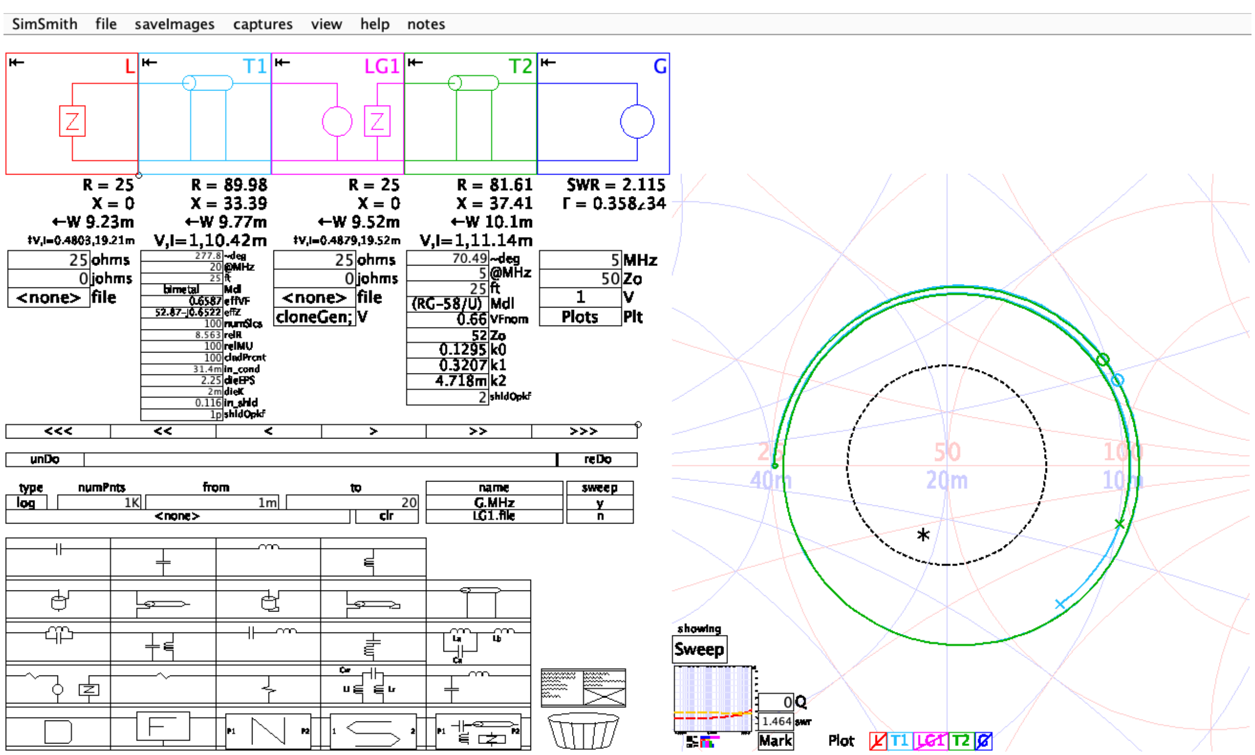


The dc resistance computed is 9.225 ohms/meter… Belden indicates 10. Belden indicates high frequency inductance is 80 nH while this model indicates 76.69 nH.

Again, there is some concern about the exact specification. For example, changing the diameter of the center conductor to .0314 inches yields the chart:



So… the model is more or less consistent with Belden published information. How does it compare to the SimSmith internal Belden 9201 model?



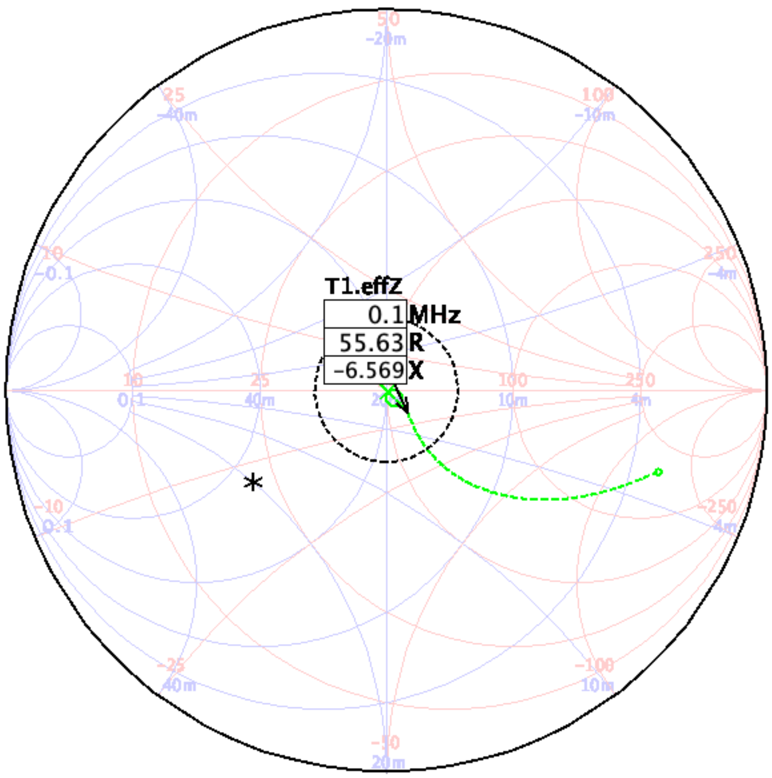
As can be seen, there is some divergence between the SimSmith internal database and the discrete model. Not sure how to interpret that just yet.

Another quick check is to compare the 'internal inductance' of a wire which should be around 50 nH meter. Subtracting the low frequency and the high frequency values of Lin is about 16nH. Converting to meters, this yields about 52 nH… again, pretty close to theory. (The difference may be due to the fact that the shield is included in this calculation.)

Other Observations

Several other interesting plots are possible. For example, here is a plot of the Zo from 1kHz to 500 MHz.

It can be seen that at frequencies below 100 kHz, the effective Zo starts to diverge significantly from the specified 52 ohms. Here it is zoomed:



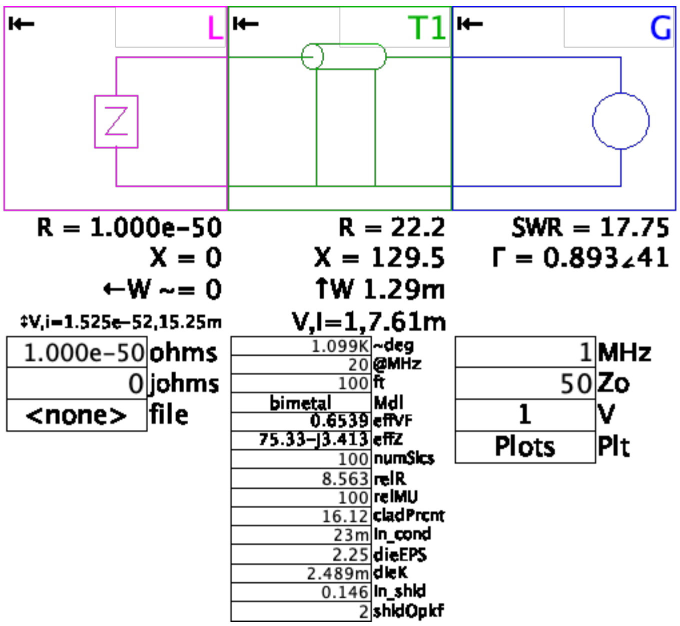
Application to Belden 8241

This is Belden's RG59 type cable. It is a solid copper clad steel conductor. The basic parameters are:

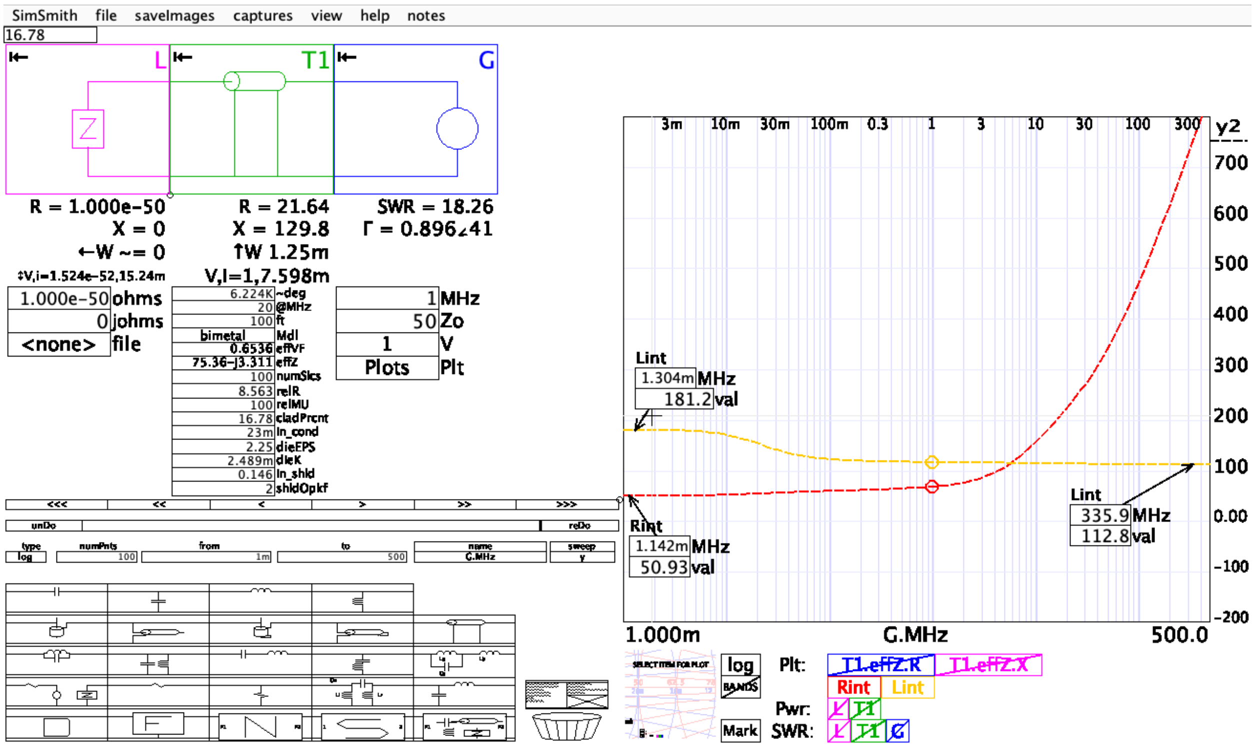
23 gauge	.023 inches

2.24	polyethylene
Shield	.146 inches
Rdc	49 ohms/ft
Lint	131 nH/ft
Cint	20.5 pF/ft

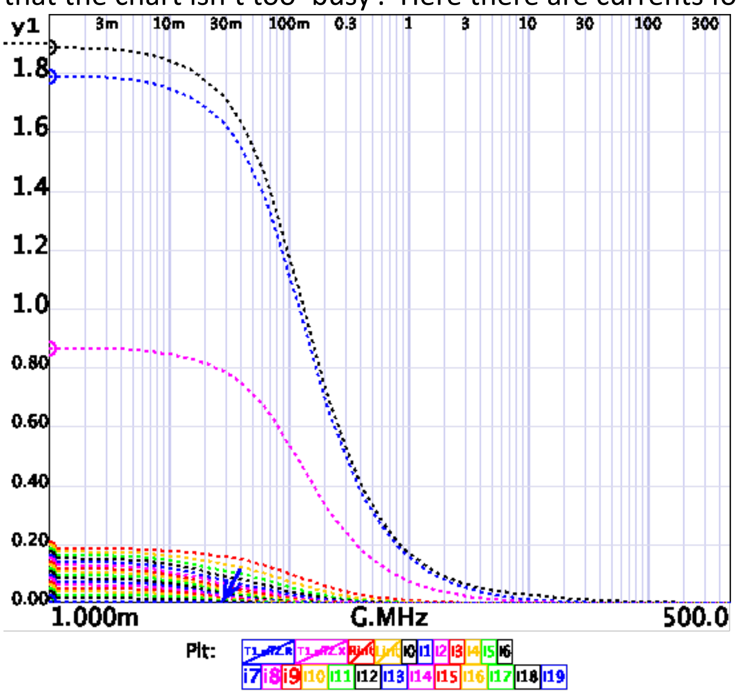
Putting this information into the model we get:



I have been unable to find any solid data concerning the steel used in this type of cable. Casting about the Internet I ultimately chose 'low carbon steel' which typically has around 9 times the resistivity and 100 times the permeability of copper. I emphasis this choice is completely unsubstantiated but is representative of a variety of steel formulations. Further, the manufacturer does not specify the thickness of the copper cladding. However, it does specify the Rdc. Adjusting the copper cladding thickness brings the reported resistance to the desired value of 49 ohms (note there are two ohms due to the shield).

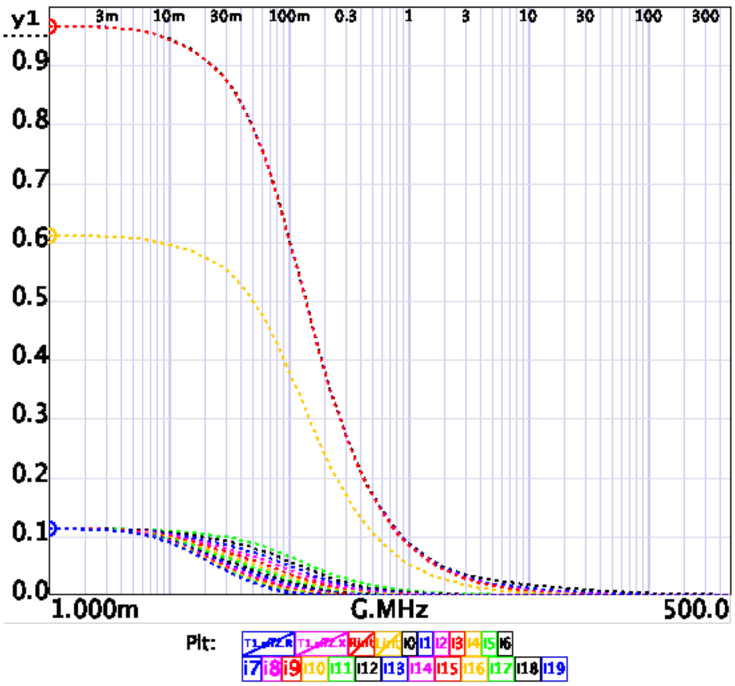


In order to examine the low frequency behavior, it is helpful to reduce the number of slices so that the chart isn't too 'busy'. Here there are currents for 20 tubes…



Notice that even at 1kHz the currents are flowing only in the outermost two to three tubes. The current clearly prefers the outermost 'copper' simply because of cross-sectional area and resistivity.

In the above the case, the tube thicknesses are all the same. SimSmith provides a second slicing mechanism where the cross-sectional area of each tube is the same. Here is the result:



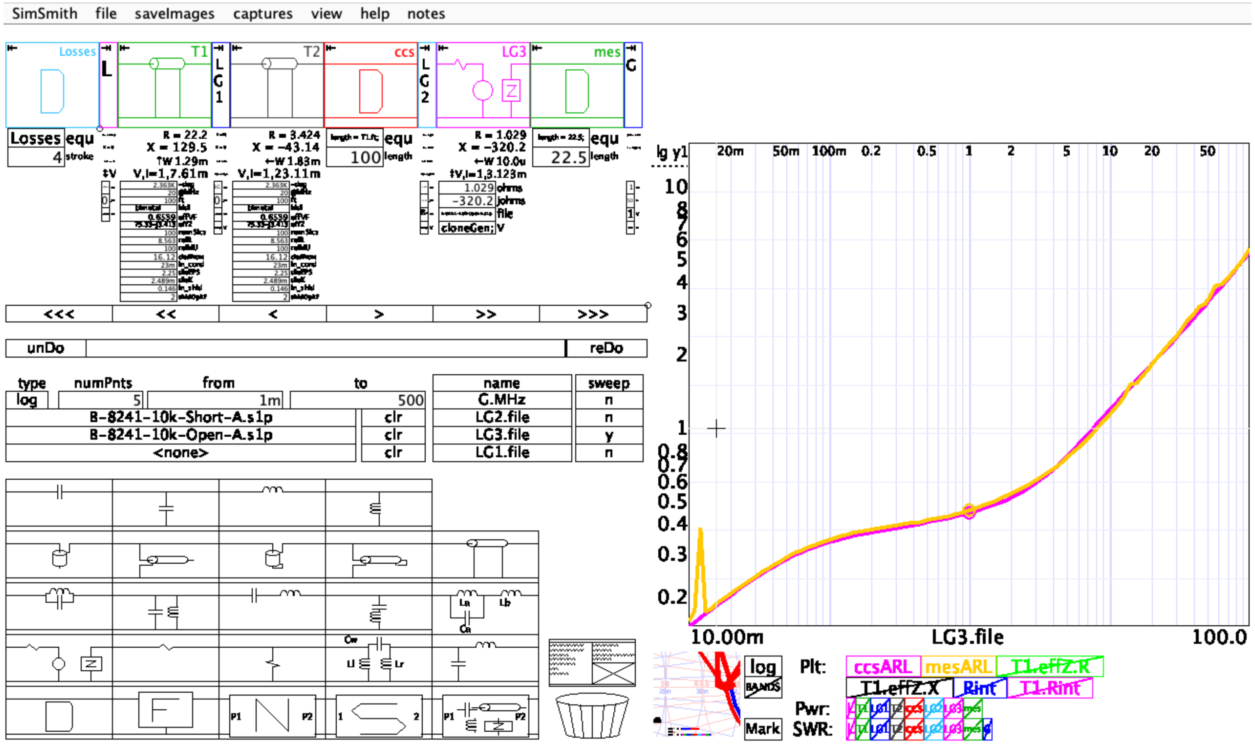
Notice that at low frequencies, there are two groups of currents. The 'copper' group with the higher currents and the steel group with lower currents. (The odd yellow trace in the above charts occurs because one of the tubes is part copper and part steel.)

Losses in 8241 'Copper Clad Steel'

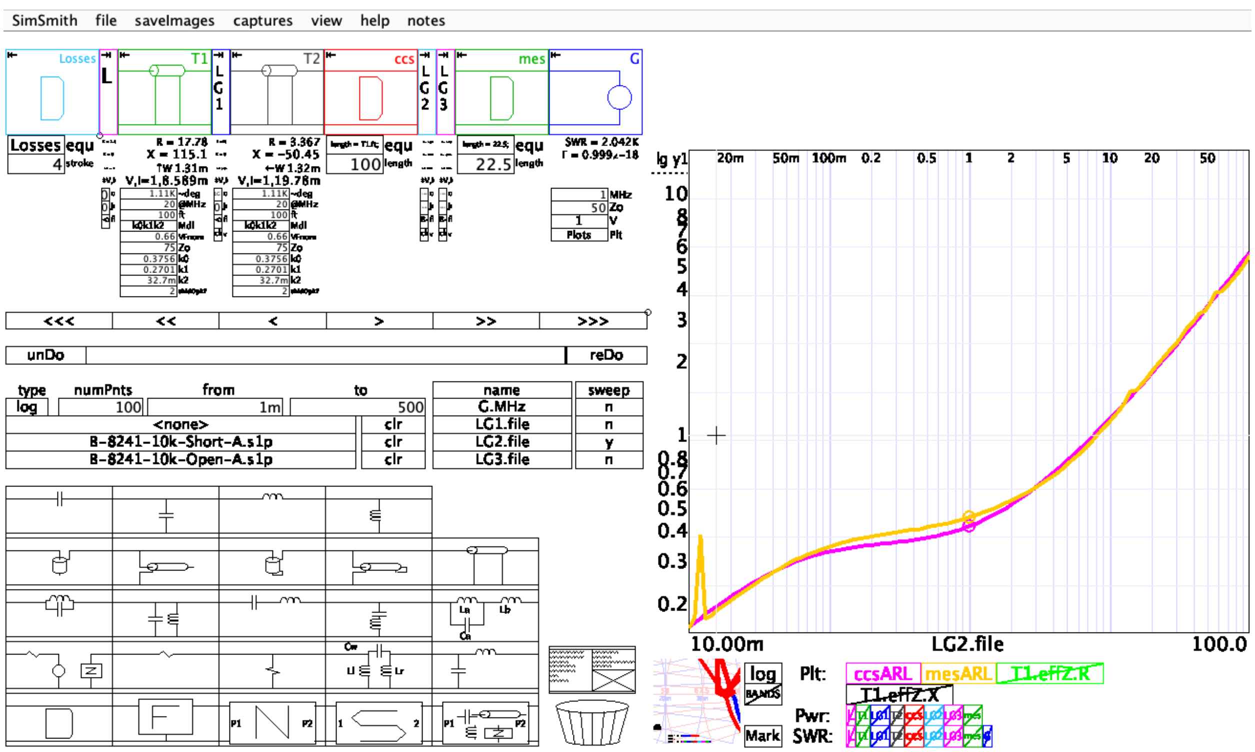
The discussion now turns to computing the losses predicted by this model. For the following, the tube thicknesses have been adjusted to provide a 'constant cross-sectional area'. This has the benefit of substantially improving the quality of high frequency analysis. (Because the outer rings are thinner.)

Additionally, the losses of the dielectric can be specified using 'dieK'. This is used to set the 'G' part of the telegrapher's equation. It has the same effect as the familiar 'k2' BUT IS NOT THE SAME VALUE AS 'k2'.

The following analysis uses Larry Benko's (W0QE) implementation of Chipman's technique for determining transmission line parameters. The 'open' and 'short' measurements were provide by KN5L. The code for the 'Losses' block is included at the end of this paper.

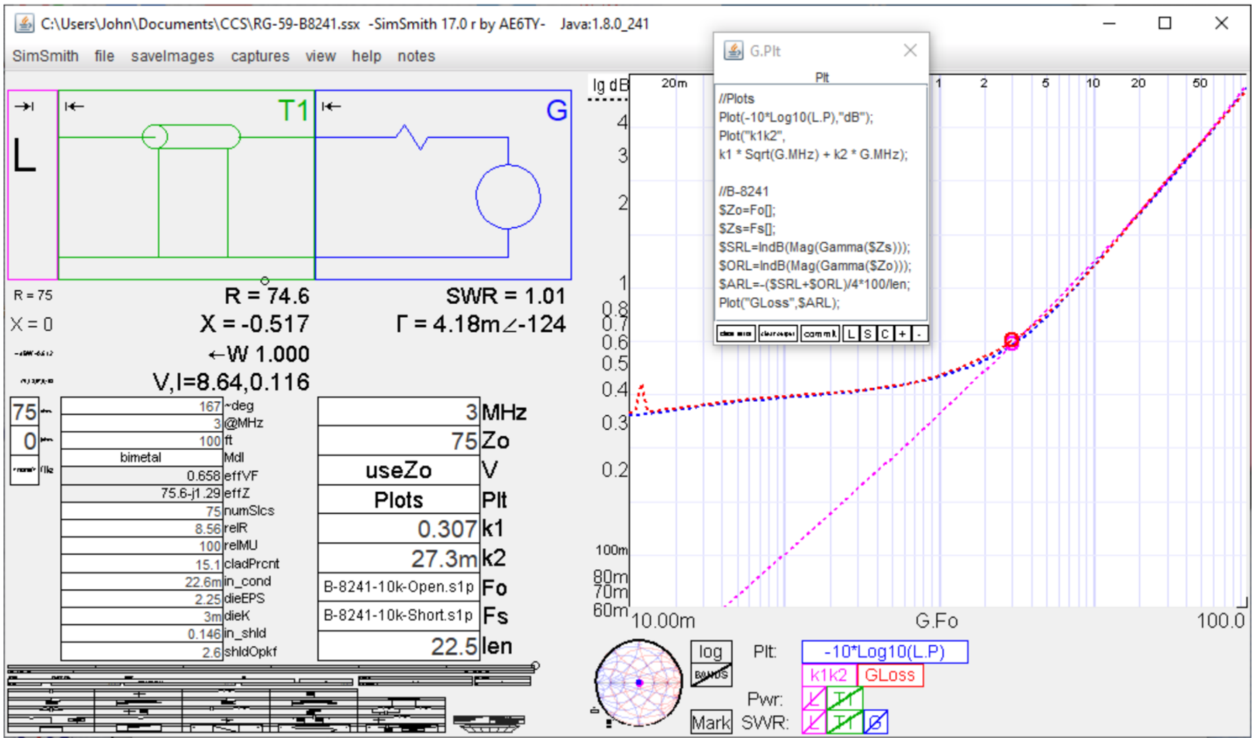


Just for reference, look at a hand tuned version of SimSmith's k0k1k2.



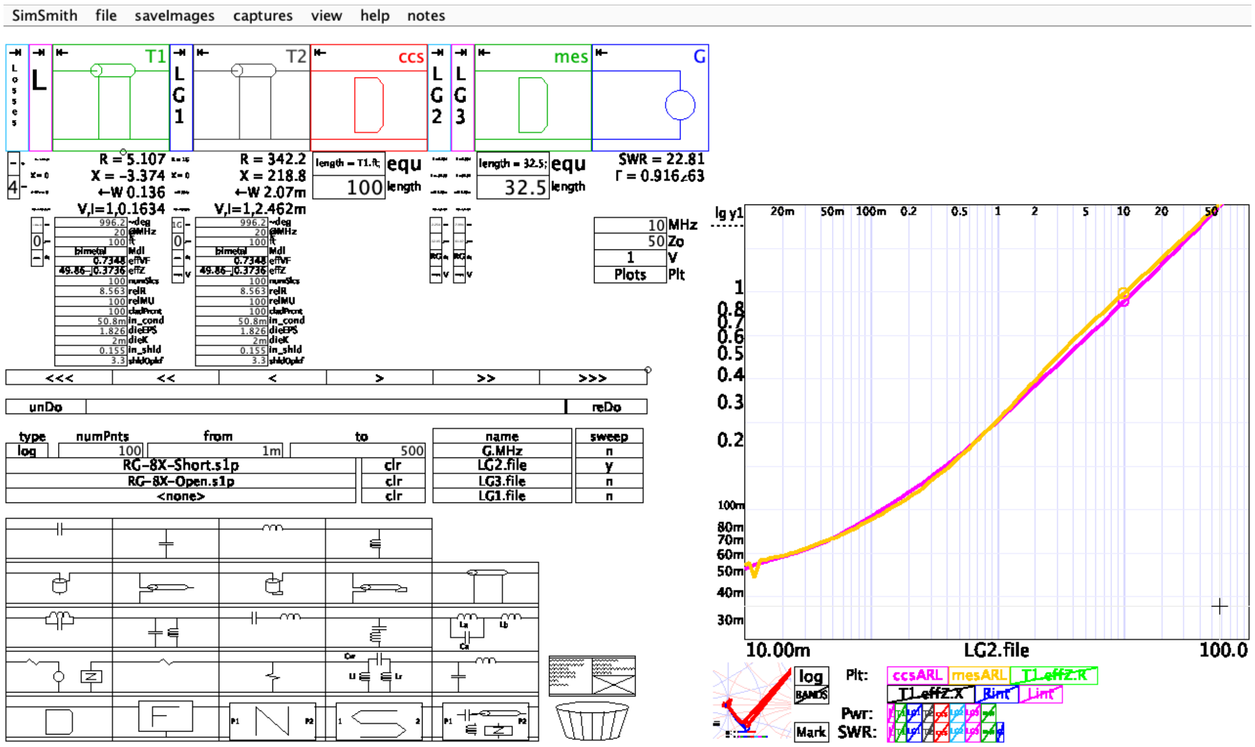
Should use Dan Maguire's (AC6LA) k0k1k2 optimizer….

John Oppenheimer (KN5L) was kind enough to do some measurements of 8241 and compare them to an updated version the bimetal model (build 17.0 r). Here is a comparison of the bimetal model and his measurements:



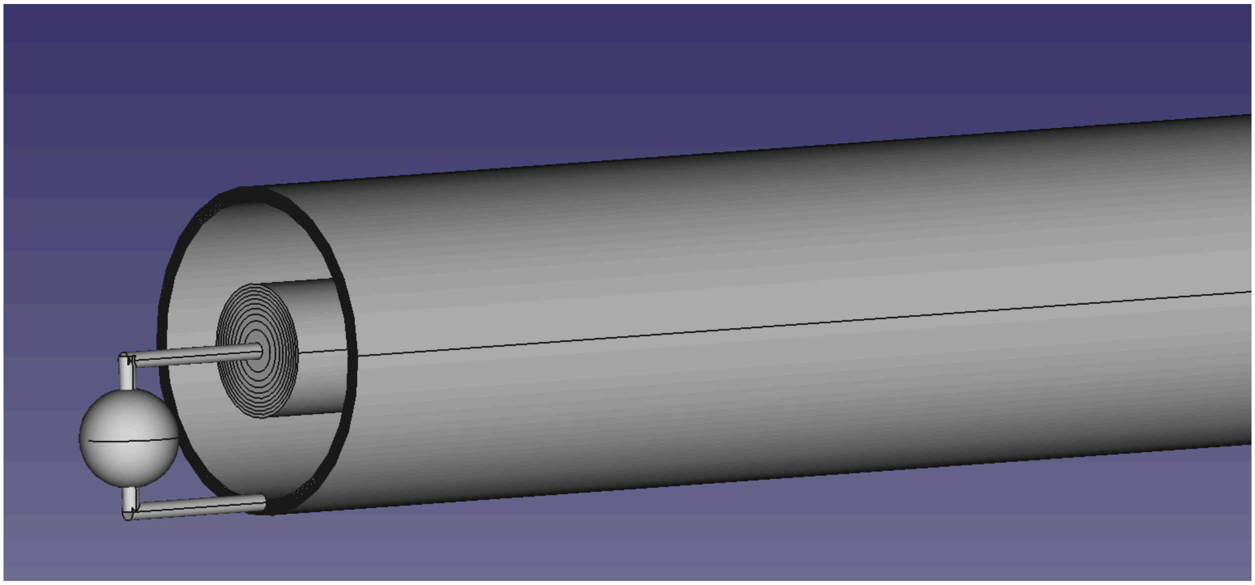
Losses Belden 9258 (RG-8x)

Just for completeness, the losses in pure copper conductor can also be examined. Note that the center conductor for this product is stranded. There are some correction factors which are not and may need to be applied. I don't know if they are significant…



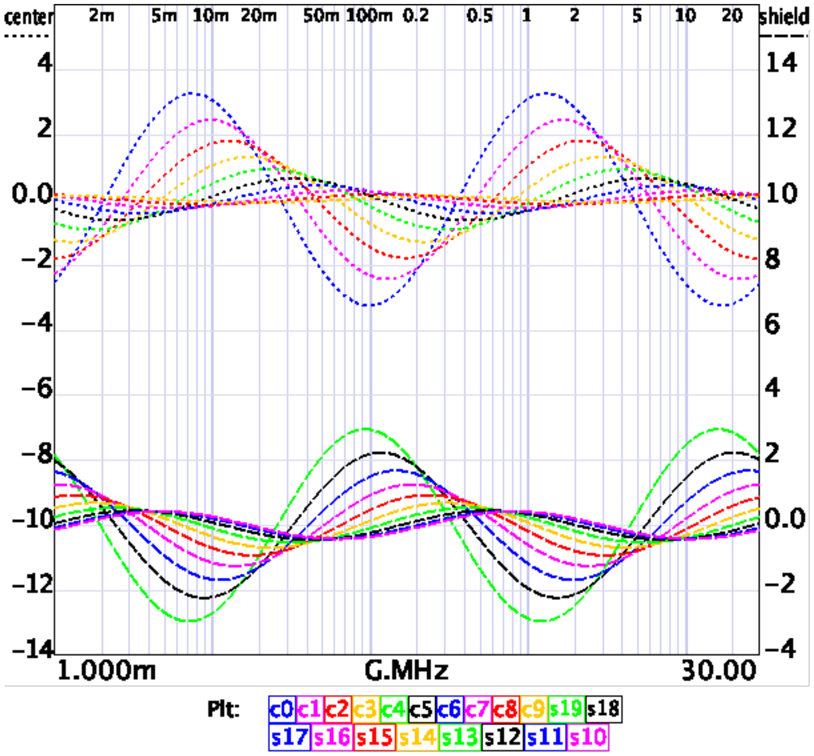
Extension to Shield

The decomposition of the center conductor into a set of concentric tubes can be extended to include the shield. There are a few caveats to be remembered. First, at present, it is assumed the shield is a solid copper tube. The shield starts at 'in_shld' which is the diameter of the shield. The thickness of the shield is computed from resistance given in ohms per 1000 feet The configuration of the tubes and 'evaluation circuit' can be visualized as shown below. (The shield 'tubes' are too small to see in the above drawing…)

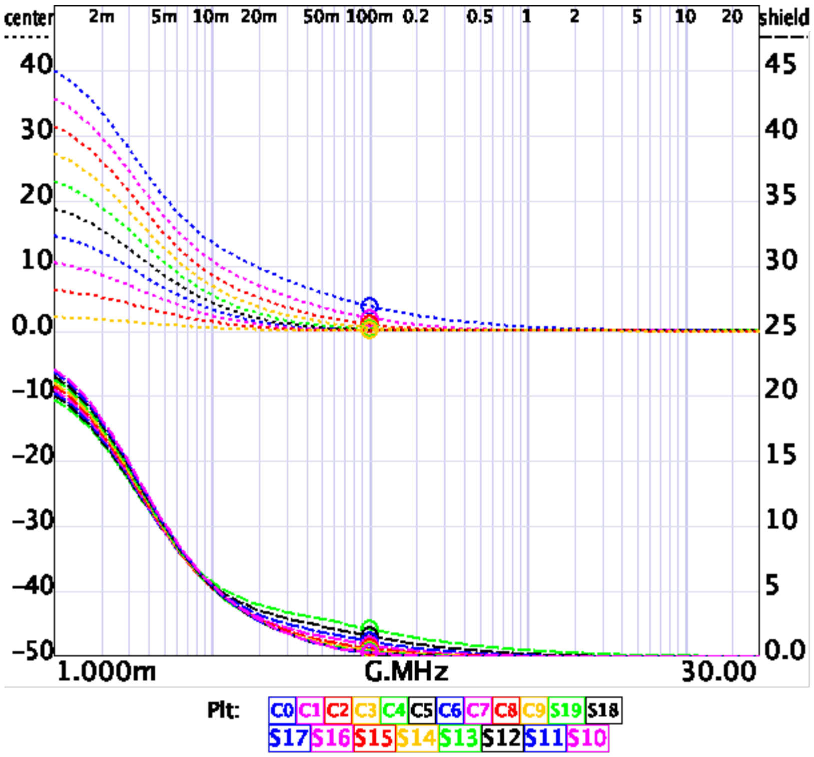


Where the sphere is a perfect current source and the other end of the coax is a perfect short. The result of the matrix solution is a voltage at the input to the center conductor and the currents through each of the tubes. The voltage is used to compute the overall impedance of the circuit which is always something of the form R+jX where X is always positive (unless one sets the dieK very high.)

The tubes in the shield all have the same thickness and half of the slices are used in the shield. As expected, there is coupling between the shield and the conductor tubes. Here is an example of plot of the currents in the conductor (top set) and shield (bottom set):



The matrix built for this analysis enforces the rule that forward current and backward current must be equal. This assumption may or may not be valid in this type of model. None-the-less, it would be a programming error if the total of currents flowing in the conductor were not equal to the total flowing in the shield. Note, however, that there is no guaranteed 1-to-1 mapping between the current flowing in the outer tube of the conductor would be equal to the current flowing on the inner tube of the shield. Here is a plot of the currents where the conductor and shield are both divided into 10 tubes each.



Examination of the shield currents leads to some interesting observations. First, the variation of currents flowing in the shield is much less than for the center conductor. Second, at low frequencies where resistance dominates any inductance, the outermost shield tube carries more current than the innermost one. This would be expected since the outer tube has more copper. Third, as the frequency increases and inductance and coupling come into play, the current starts to favor the inner layer of the shield. All things which would be expected from outside experience and gratifying to see in the model.

Computation of Transmission Line Characteristics

The telegrapher's equations compute Zo using the formula:

Zo = Sqrt((R+jwL) / (G + jwC))

For any given frequency, solution of the preceding matrix delivers R+jwL. The term 'G' is computed from freq*dielectricK as given by the user and C which is computed from the shield spacing and the relative permittivity as provided by the user.

The transmission coefficient, gamma, is computed from:

gamma = Sqrt((R+jwL) * (G+jwC))

and the velocity factor is simply the inverse of the imaginary part of gamma scaled appropriately:

VF = omega/speedOfLight / Imag(gamma)

Extension to Window Line (like 551)

Of particular interest in this project was the extension of these concepts to the commonly used window line called “551”. This product is billed as a 450 ohm product but is, in reality, only a 400 ohm product.

551 window line is constructed using 18 gauge, 21% copper clad steel conductors spaced approximately .75 to .8 inches apart. There is a plastic spacer between the conductors with windows cut in it for various reasons.

This product is of interest because it is widely used in the amateur radio community, particularly when driving non-resonant dipoles. Often, it is used as a ‘low loss’ way to drive dipoles well below resonance so as to reach the 80 or even 160 meter band using a transmatch tuner. Understanding the limitations of 551 might prove useful.

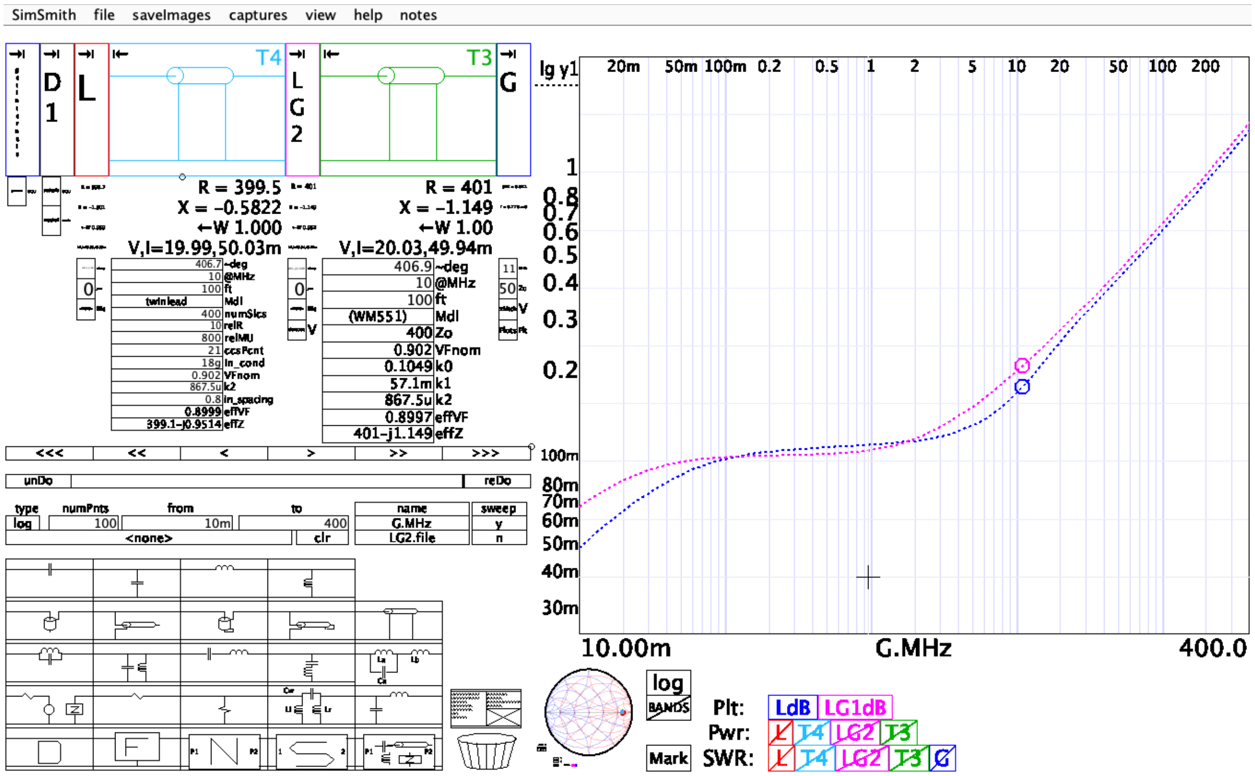
As it turns out, the above computational techniques can be used to model window line with high confidence. There are two specific parameters which require adjustment: the computation of inductance, and the determination of the effective relative permittivity.

The effective permittivity is difficult to calculate from basic geometry. However, the it is relatively easy to determine given the velocity factor of the product. In the absence of significant permeability, the velocity factor of a transmission line is dominated by the permittivity using the equation:

VFnom = 1/Sqrt(relEPS);

Consequently, given the manufacturer’s nominal velocity factor ‘VFnom”, the relEPS can be computed as: (1/VF) ^2.

In practice, the ‘twinlead’ model actually computes the parameters of a transmission line consisting of a single conductor above a ground plane. The formulae for computing inductance and capacitance of this configuration are well published and surprisingly familiar. Having computed the transmission line parameters for this single conductor above a ground plane, the model simply scales the transmission line parameters. Thus, the values for R and L are doubled and those for G and C are halved. A comparison of this model with existing k0k1k2 models is:



A few notes of interest are: the 'twinlead' model parameters were taken from the manufacturer's dimensions and a guess as to the relMU and relR. The resulting effVF and effZo are very close to those established by independent measurement and tunig of the k0k1k2 model.

Of particular interest is the exact shape if the blue line (the 'twinlead' model) when compared to the k0k1k2 magenta model. Note that on the far-left part of the graph, the twinlead model has lower losses than the k0k1k2 model. This is because the internal conductance of the steel is coming into play. An even more interesting phenomena is that the twinlead model predicts that losses stay lower than predicted at the onset of the skin effect. This represents the area in which Johnson's 'smoothing' technique (used to avoid the Bessel functions) shows its weakness.

Missing Considerations

There are many 'variables' this model does not incorporate or could be refined. A short and by no means comprehensive list is:

Stranded vs solid center conductors
Shield braid density
Shield composition… foil, materials, etc.

Steel properties

Reminders

This project is an ongoing effort. Each release of the model will have improvements and possibly introduce bugs. Parameters to the model will be added, renamed, removed and redefined. For this reason, the above charts probably cannot be reproduced exactly.

In many ways, the proper interpretation of this paper is… 'huh'.

Summary

Johnson, section 2.9 describes a 'Concentric-Ring Skin-Effect Model'. This paper presents a concrete implementation of the concepts there-in. The primary impetus for this implementation was the exploration of bimetallic center conductors, specifically, Copper Clad Steel.

The model proposed by Johnson is derived from the basic laws of electrodynamics; Ohm's Law, Ampere's Law, and Faraday's Law. The numeric analysis of this model tracks quite well with his analytic model summarized as the 'k0k1k2' model.

Perhaps the true bottom line is, to a very reasonable approximation, the use of a k0k1k2 model is justified for nearly all coaxial cable types for a very reasonable approximation.

Losses Code

The 'Losses' block was originally written by Larry Benko (W0QE) and modified to make it a function. The losses block code is:

```
//Losses
stroke;
dcl losses(comp,Source1,Source2,feet) {
	if (stroke == 0)
		stroke = 4;

	dcl Src1 = Source1.Z;
	dcl Src2 = Source2.Z;
	dcl Zo = Sqrt(Src1*Src2);

	// from Larry's video
	// specify reference frequency for Gamma explicitly.
	dcl SRL = -IndB(Mag(Gamma(Src1,Zo)));
	dcl ORL = -IndB(Mag(Gamma(Src2,Zo)));

	dcl ARL = (SRL+ORL)/2 /2 * 100/feet;
	Smith(comp.color,Stroke(stroke),comp.LOC+"Zcalc",Zo);
	//Plot(Stroke(stroke),comp.LOC+"SRL",SRL);
	//Plot(Stroke(stroke),comp.LOC+"ORL",ORL);
```

```
    Plot(comp.color,Stroke(stroke),comp.LOC+"ARL",ARL);
}
```

Note that the above code computes the losses AND provides two plotting functions: a 'Smith' plot of the computed reference impedance Zo, and a 'Plot' of the 'averaged' loss of the open and short sources. Both these plots use the name of the calling block specified by 'comp' as well as the color of that component. Thus, the lines on the charts have names and color controlled by the calling component so as to differentiate them. (The color is available only with up to date builds of version 16p8. Otherwise, you'll have to edit out the color part.)

This code is invoked in a D block called something like 'ccs'. An invocation might be:

```
Losses.losses(this,T1,T2,T1.ft);                // take length from the transmission line.
```

Or

```
Losses.losses(this,LG2,LG3,length=22.5);        // display length
```

References

Johnson, Howard & Graham, Martin. "High-Speed Signal Propagation, Advanced Black Magic". 2003 Pearson Education, Inc. Published as Prentice Hall Professional Technical Reference.

Chipman, Robert. "Theory and Problems of TRANSMISSION LINES". 1968 Schaum's Outline Series, McGraw-Hill Book Company

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